Journal of Pipelines, 6 (1987) 319-323
Elsevier Science Publishers B.V., Amsterdam — Printed in the Netherlands

A METHOD TO MEASURE DISCHARGE IN PIPELINES

L. Flint-Petersen and N. Rajaratnam
Department of Civil Engineering, University of Alberta, Edmonton, Alberta T6G (Canada)

(Received June 30, 1986; accepted March 25, 1987)

SUMMARY

For fully developed turbulent flow, in smooth or rough, straight circular pipes, using one velocity reading at any distance from the boundary by a velocity measuring instrument (MSR Magflowmeter, pitot-static tube or any other similar device), the mean velocity and hence, flowrate, can be directly computed using a method presented in this paper.

NOTATION

d	diameter of pipe
f	Darcy–Weisbach friction factor
k_s	Nikuradse equivalent sand roughness
Q	volumetric flow rate
R	Reynolds number, equal to Vd/ν
R_u	Reynolds number, equal to ud/ν
r_0	radius of pipe
U	value of axial velocity on axis of pipe
u	axial velocity at a normal distance y from wall
u_*	Prandtl shear velocity
V	mean velocity of flow
y	normal distance from the wall
α	ratio of u to V
β	ratio of y to r_0
ν	kinematic viscosity of fluid
ρ	mass density of fluid
τ_0	wall shear stress

INTRODUCTION

The measurement of volumetric flowrate or discharge of fluids flowing in pipelines is important in municipal and industrial installations. As a result, a number of methods have been developed for measuring the discharge [1,2]. In this paper, we confine our attention to the turbulent motion of Newtonian liquids in straight circular pipes. A recent development in this field is the MSR Magflowmeter [3] which measures the velocity at a point in the pipe. This meter is rugged and is ideally suited for use in large pipelines carrying a wide range of liquids. This meter is generally placed with its sensor at a radial distance of 0.22 r_0 (where r_0 is the radius of the pipe) from the pipe wall (or 0.78 r_0 from the axis of the pipe). In this position, the time-averaged axial velocity u measured by the meter is equal to the mean velocity V. The mean velocity V is, of course, equal to the discharge Q divided by the cross-sectional area of the pipe. This value of 0.22 r_0 from the wall is slightly less than the value of 0.24 r_0 obtained by Aichelen from Nikuradse's measurements [1]. The value of 0.22 r_0 is obtained based on the idea that the axial velocity profile in fully-developed turbulent flow in a straight circular pipe with a smooth or rough wall is well-described by the defect law [4]

$$\frac{U-u}{u_*}=5.75 \log \frac{r_0}{y} \tag{1}$$

In eqn. (1), u is the time-averaged axial velocity at a radial distance of y from the wall, U is the maximum value of u occurring on the axis of the pipe and u_* is the Prandtl shear velocity, equal to $\sqrt{\tau_0/\rho}$ where τ_0 is the wall shear stress and ρ is the mass density of the fluid [4]. Equation (1) is valid from the axis of the pipe to y/r_0 as small as 0.02 [4.5].

Let us now assume that for some reason, the velocity measuring device (which could be a pitot-static probe or current meter or Magflow-meter) is placed at a distance of y from the wall which is not equal of 0.22 r_0. Let u be the velocity at this probe location. We are interested in predicting the mean velocity V from the observed local velocity u. To perform this operation, for turbulent flow in a circular pipe, smooth or rough, we can write eqn. (2) as follows [4]:

$$V=U-3.75\, u_* \tag{2}$$

Schlichting [4] suggests changing 3.75 to 4.07 to agree with Nikuradse's experiments for smooth pipes, whereas for rough turbulent flow the value of 3.75 is retained. If we adopt this procedure, in any general plot such as the Moody diagram, or the diagrams we are going to present later in this paper, the curves for different relative roughnesses do not join the smooth turbulent flow line asymptotically, but cut across it. Further, the difference between using either 3.75 or 4.07 in our calculations was less than about 3% for smooth turbulent flow. Hence we retain the value of 3.75.

Let $u=\alpha V$ and $y=\beta r_0$. We can rewrite eqn. (2) as

$$\frac{V\,(1-\alpha)}{u_*}+3.75=5.75 \log \frac{r_0}{y} \tag{3}$$

To predict V or α from eqn. (3), we have to use a trial and error method [3′]. We present, herein, a direct method of solution for this problem for turbulent flow in smooth or rough circular pipes.

THEORY OF THE METHODS AND CHARTS

Turbulent flow in smooth and rough pipes can be divided into three regimes of smooth turbulent, rough turbulent and transition from smooth to rough turbulent regimes [4]. For smooth turbulent flow, the Darcy–Weisbach friction factor f, which is related to V by the relation

$$\frac{V}{u_*}=\sqrt{\frac{8}{f}} \tag{4}$$

is given by the Prandtl equation [4]

$$\frac{1}{\sqrt{f}}=2 \log \mathrm{R} \sqrt{f}-0.8 \tag{5}$$

where R is the Reynolds number equal to Vd/ν, d being the pipe diameter and ν the kinematic viscosity of the fluid. For rough turbulent flow

$$\frac{1}{\sqrt{f}}=2.0 \log\frac{r_0}{k_s}+1.74 \tag{6}$$

wherein k_s is the Nikuradse equivalent sand roughness of the pipe. The Colebrook–White equation

$$\frac{1}{\sqrt{f}}=1.74-2.0 \log \left[\frac{k_s}{r_0}+\frac{18.7}{R\sqrt{f}}\right] \tag{7}$$

describes the transition region from smooth to rough turbulent flow. Equation (7) merges asymptotically with eqn. (5) on one end and with eqn. (6) at the other end.

Let us consider the Colebrook–White equation. We can rewrite it, after some rearrangement as

$$A=1.74-2.0 \log \left[2\frac{k_s}{d}+\frac{18.7}{\frac{ud}{\nu}\frac{1}{\alpha}}A\right] \tag{8}$$

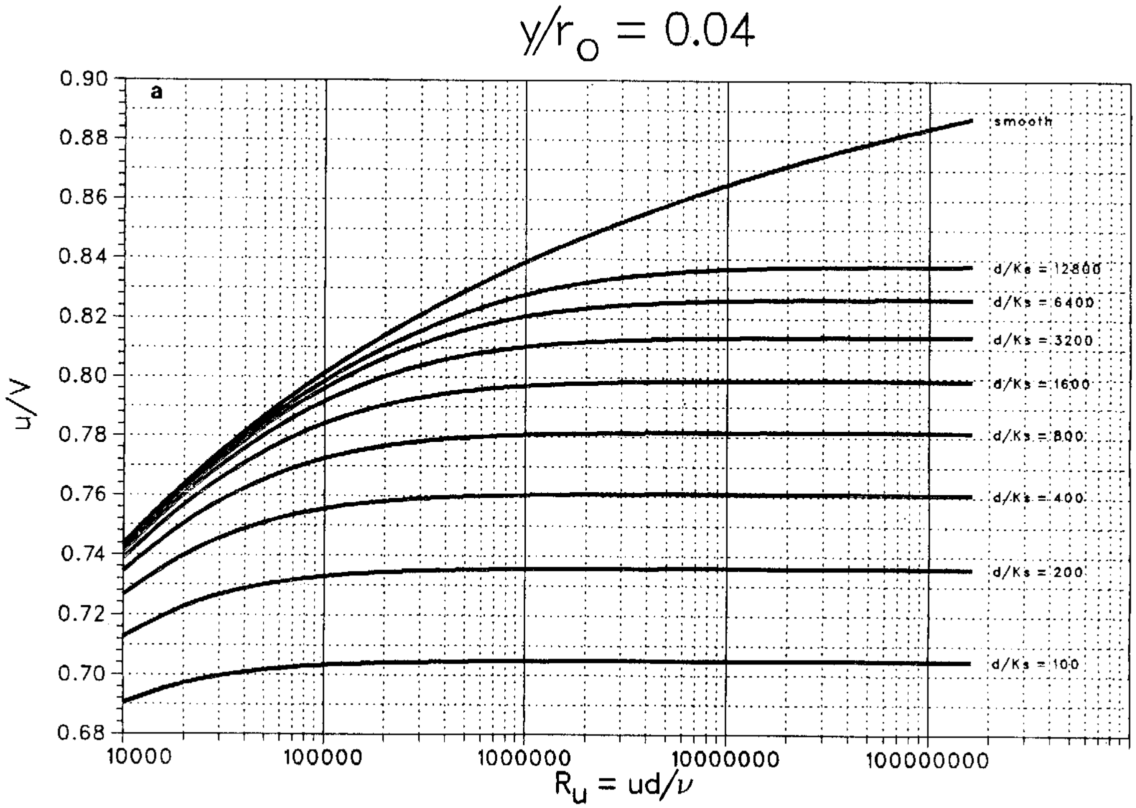

Fig. 1. Chart for predicting mean velocity from single-point velocity observation for turbulent flow in circular pipes; $y/r_0 = 0.04$.

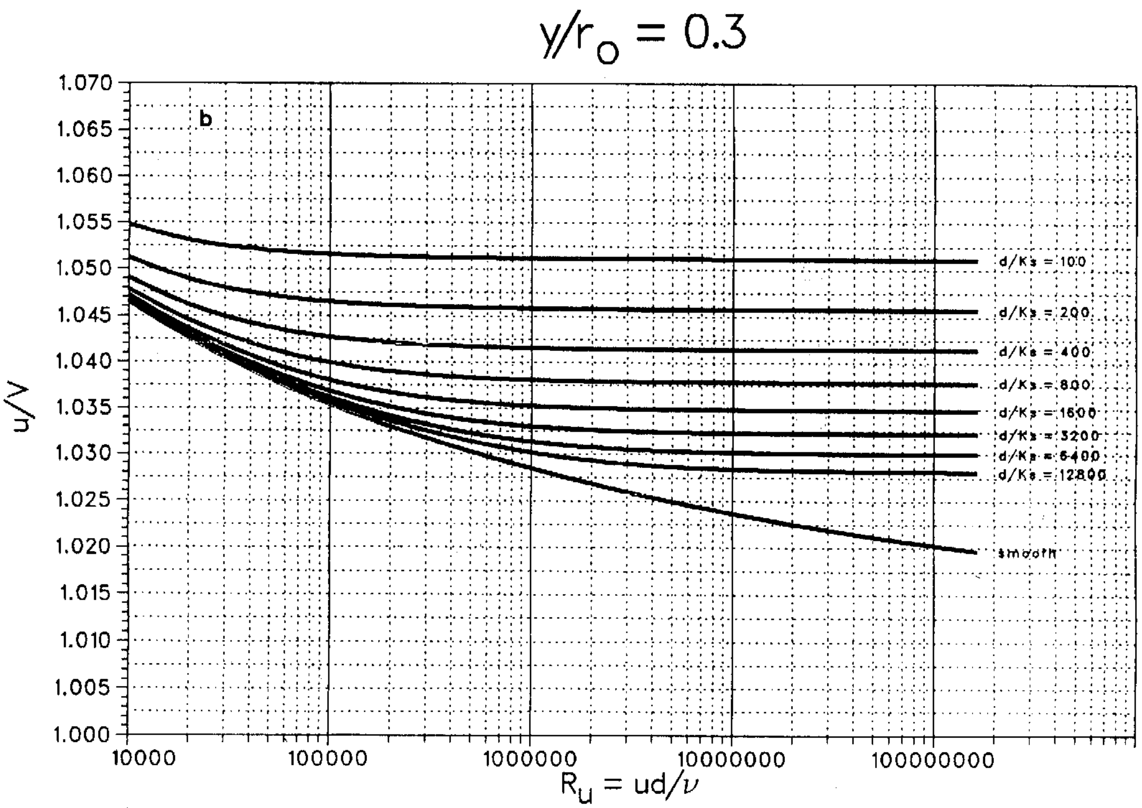

Fig. 2. Chart for predicting mean velocity from single-point velocity observation for turbulent flow in circular pipes; $y/r_0 = 0.3$.

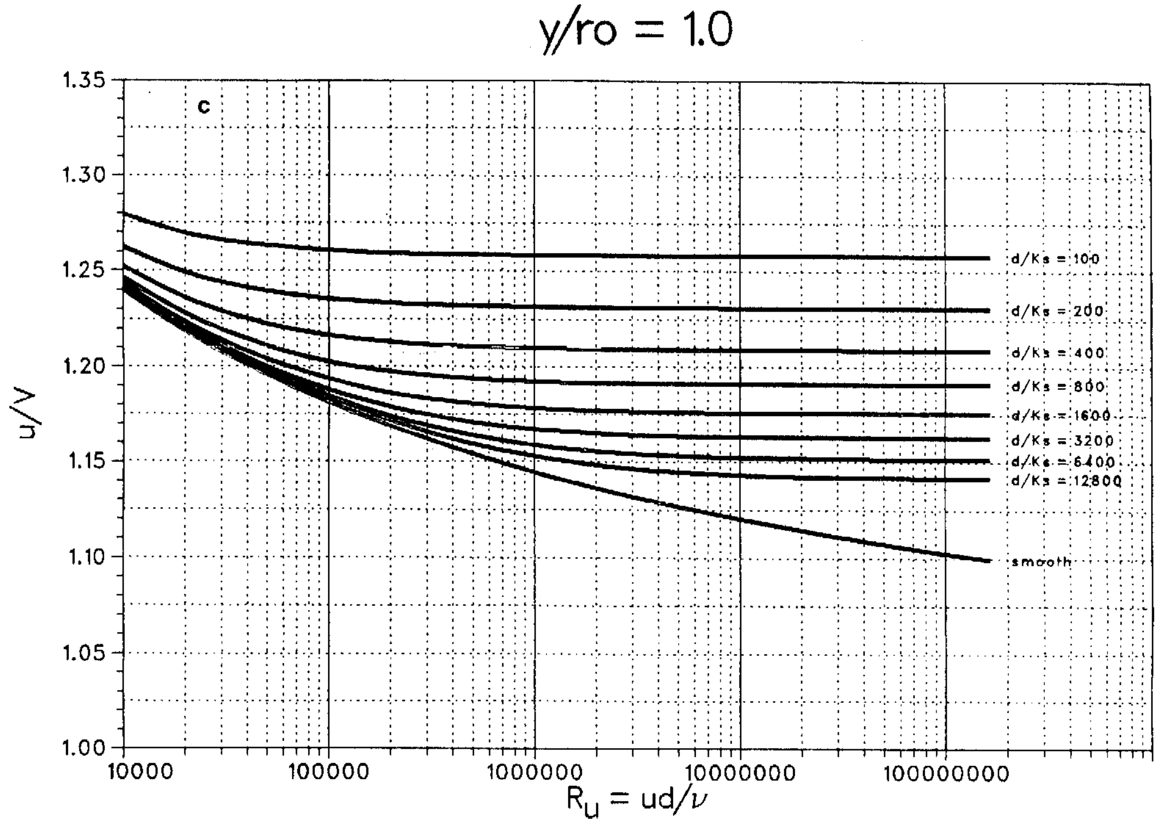

Fig. 3. Chart for predicting mean velocity from single-point velocity observation for turbulent flow in circular pipes; $y/r_0=1.0$.

where

$$A=-\left[\frac{3.75+5.75\log\beta}{\sqrt{8}\,(1-\alpha)}\right] \tag{9}$$

For a pipe of given relative roughness k_s/d, for a particular location of the velocity measuring instrument, we know β and ud/ν; the problem is to predict α. Equations (8) and (9) were solved by the method of bisection and the results can be plotted with $\alpha=u/V$ versus $R_u=ud/\nu$ for different values of the parameter k_s/d or d/k_s and $\beta=y/r_0$.

Before looking at the computed results, let us consider eqn. (8) for smooth turbulent flow. For this regime, eqn. (8) can be rewritten as:

$$A=2.0\log\left[\frac{R_u}{\alpha}\frac{1}{A}\right]-0.80 \tag{10}$$

with A defined by eqn. (9). Equation (10) was solved in a similar manner to that of eqn. (8). For practical purposes, it is not necessary to consider eqn. (6) separately as eqn. (7) predicts the rough turbulent regime adequately.

The computations were performed for $\beta=0.04$, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18, 0.20, 0.30, 0.40, 0.5 and 1.0 for a wide range of d/k_s as well as for the smooth state. Three typical sets of curves are shown in Figs. 1–3 for $\beta=0.04$, 0.3 and 1.0. These diagrams and others not reproduced herein showed the change in the geometrical configuration of these curves as β increases past 0.22. The range of R_u included increases from 10^4 to about 10^8. Using these diagrams (a more extensive set of Figs. 1–3 is available from the authors), for any position of the velocity measuring instrument (for the given β, pick the proper diagram), using the curves in the proper diagram, the mean velocity V and hence the flow rate Q can readily be found.

For this single velocity method to predict the flow rate Q correctly, the flow has to be fully developed; the pipe should be straight or have enough straight length after either bends or valves, and there should be no residual swirl in the pipe. In any situation, one precautionary measure would be to choose either two symmetric locations or compute Q with the observations from any two locations

CONCLUSIONS

A convenient method has been presented to compute the flowrate from one-point velocity observation for turbulent flow in circular pipes.

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